



# CSC212

# Data Structure

## - Section FG

# Lecture 13

# Recursive Thinking

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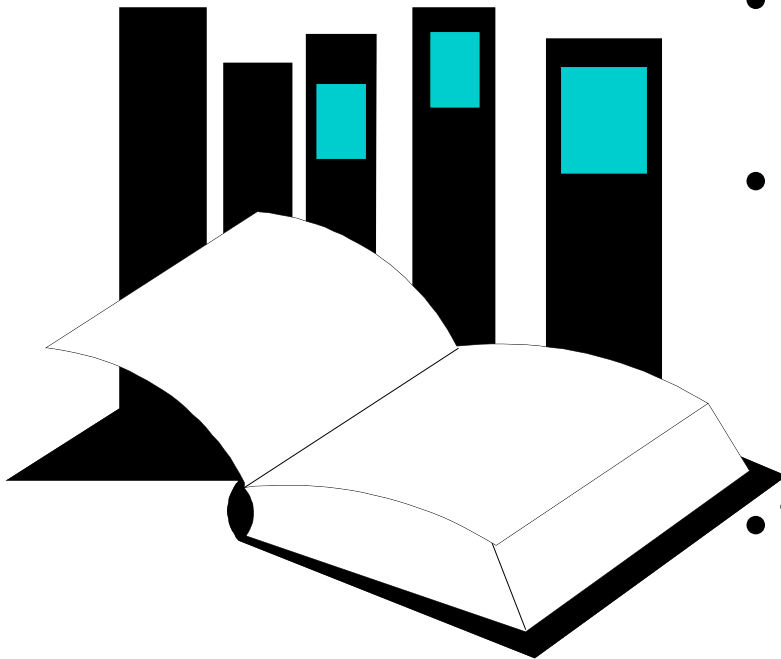
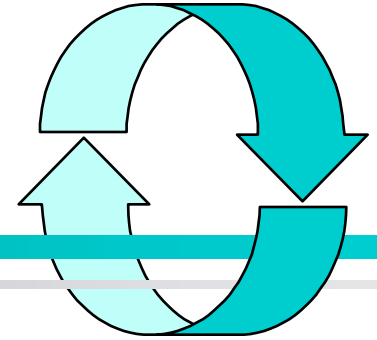
# Outline of This Lecture

- Start with an Example of Recursion
  - “racing car” – not in the textbook
  - using slides (provided by the authors)
- Recursive Thinking: General Form
- Tracing Recursive Calls
  - using blackboard to show the concepts
- A Closer Look at Recursion
  - activation record and runtime stack



# Recursive Thinking

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- Chapter 9 introduces the technique of recursive programming.
- As you have seen, recursive programming involves spotting smaller occurrences of a problem within the problem itself.
- This presentation gives an additional example, which is not in the book.

**Data Structures  
and Other Objects  
Using C++**

# The Car Racer Slides were modified from

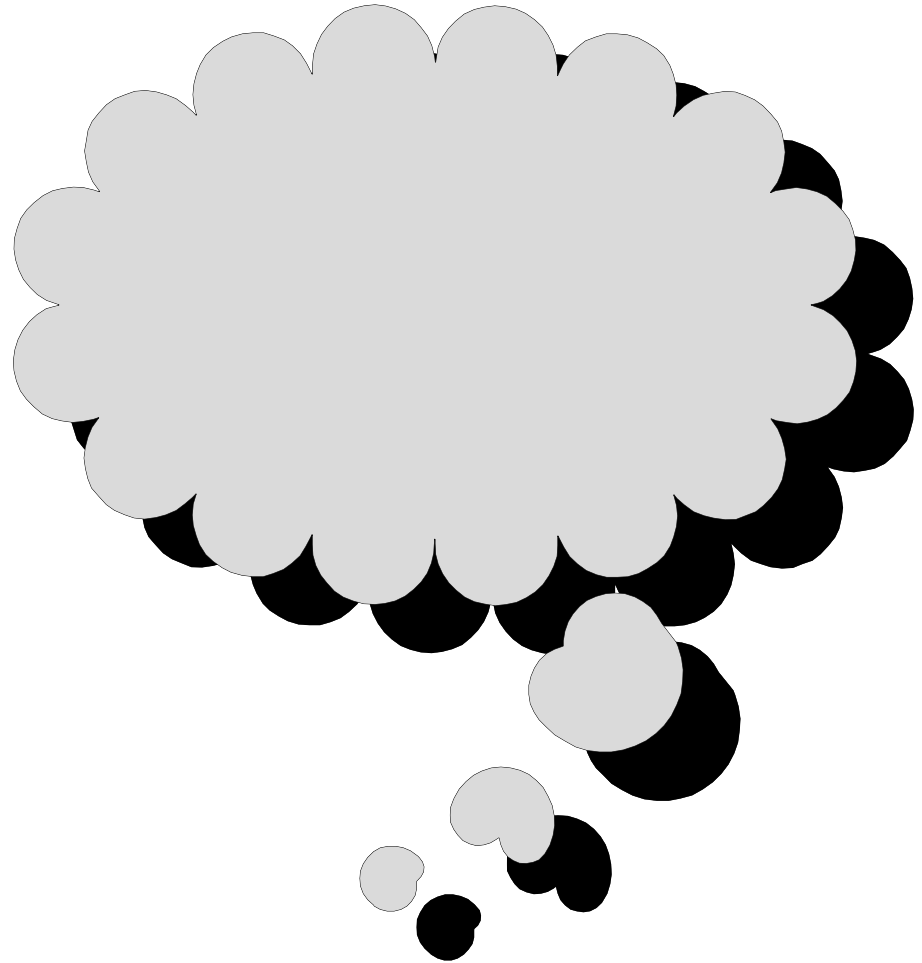
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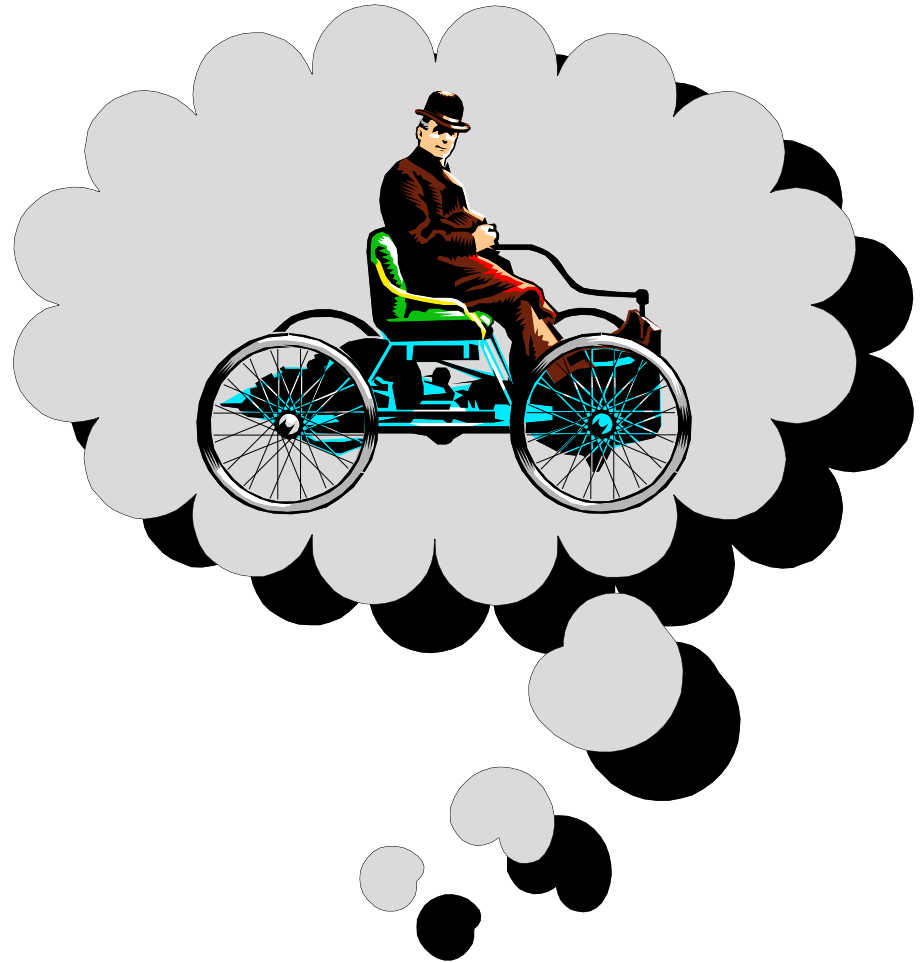
# A Car Object

- To start the example, think about your favorite family car



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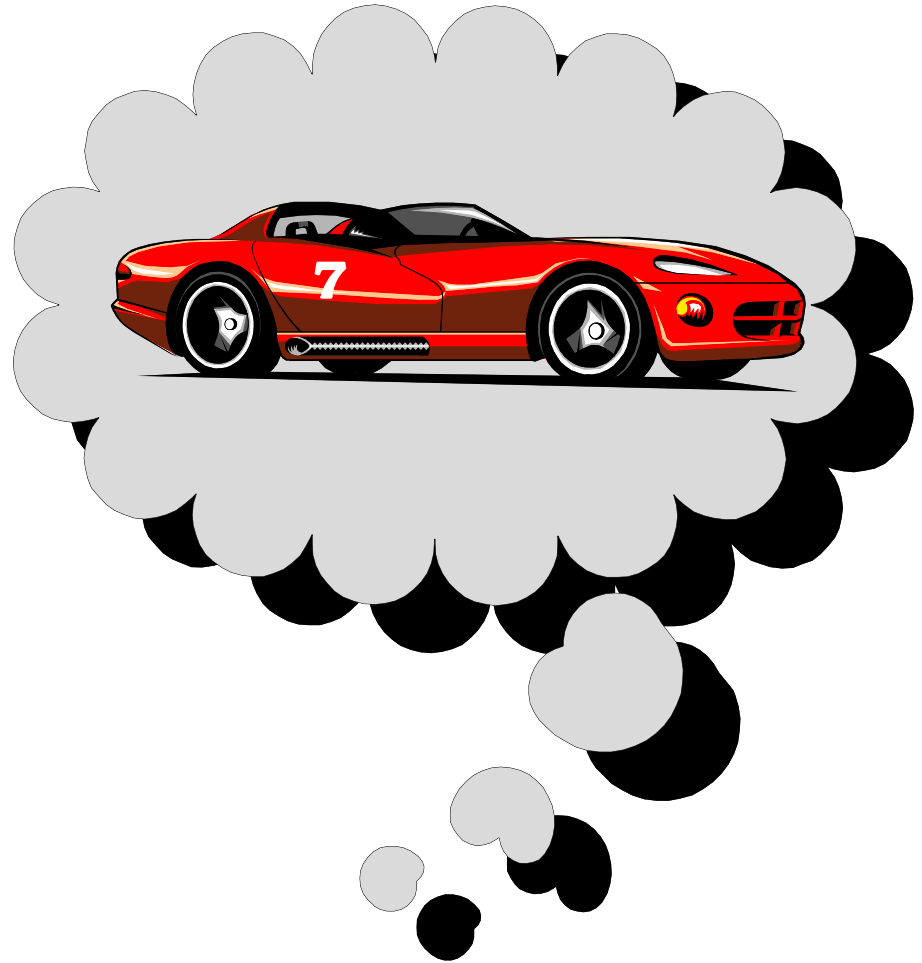
# A Car Object

- To start the example, think about your favorite family car



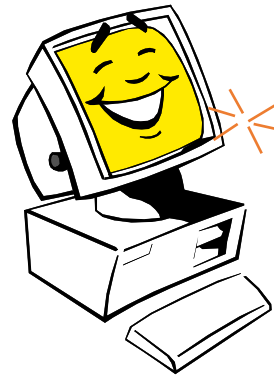
# A Car Object

- To start the example, think about your favorite family car

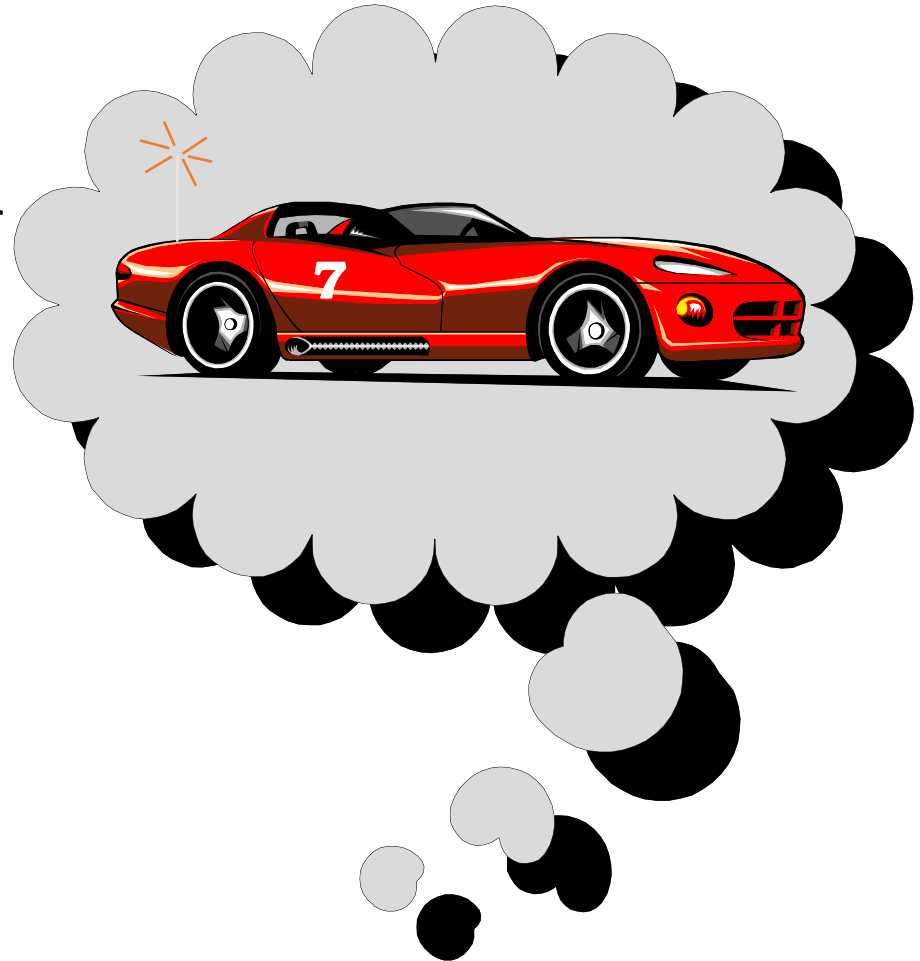




# A Car Object



- To start the example, think about your favorite family car
- Imagine that the car is controlled by a radio signal from a computer



# A Car Class

- To start the example, think about your favorite family car
- Imagine that the car is controlled by a radio signal from a computer
- The radio signals are generated by activating member functions of a Car object

```
class Car  
{  
  public:  
    ...  
};
```

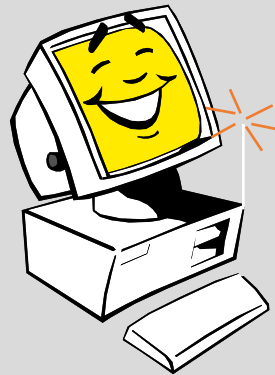
A diagram consisting of a light gray rectangular box with a black border. Inside the box, the text 'class Car' is followed by an opening curly brace '{', the word 'public:', three orange dots '...', and a closing curly brace '}' followed by a semicolon ';'. An orange arrow with a black outline starts from the left side of the box, curves upwards and to the right, and points directly at the closing curly brace '}' of the class definition.

# Member Functions for the Car Class

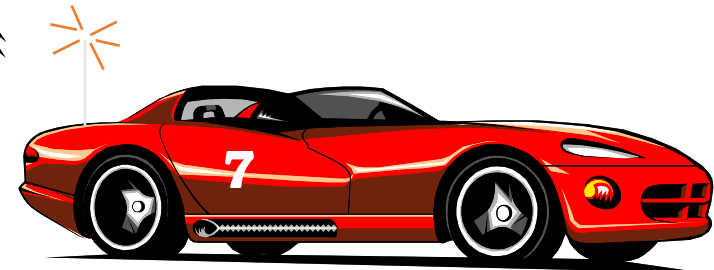
```
class Car
{
public:
    Car(int car_number);
    void move( );
    void turn_around( );
    bool is_blocked( );
private:
    { We don't need to know the private fields! }
    . . .
};
```

# The Constructor

```
int main( )  
{  
    Car racer(7);  
  
    . . .
```



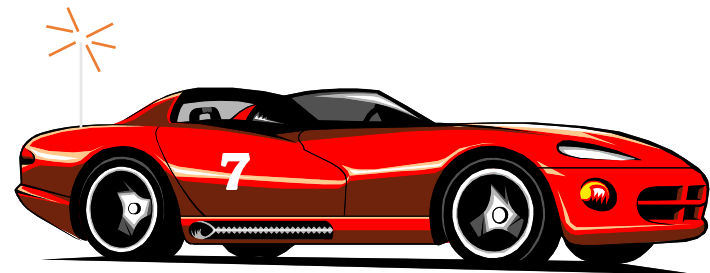
When we declare a Car and activate the constructor, the computer makes a radio link with a car that has a particular number.



# The turn\_around Function

```
int main( )  
{  
    Car racer(7);  
  
    racer.turn_around( );  
  
    . . .  
}
```

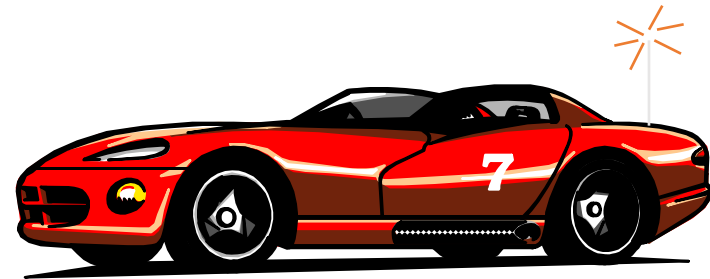
When we activate turn\_around, the computer signals the car to turn 180 degrees.



# The turn\_around Function

```
int main( )  
{  
    Car racer(7);  
  
    racer.turn_around( );  
    . . .
```

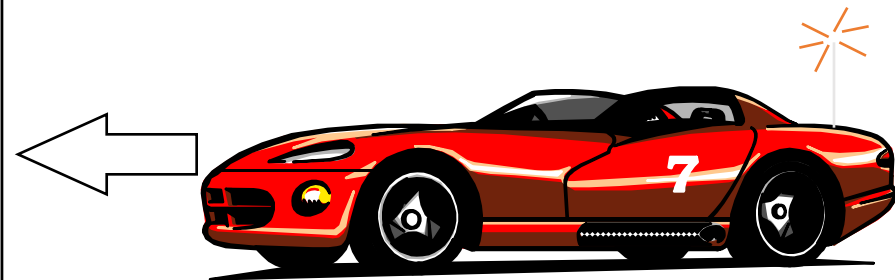
When we activate turn\_around, the computer signals the car to turn 180 degrees.



# The move Function

```
int main( )  
{  
    Car racer(7);  
  
    racer.turn_around( );  
    racer.move( );  
    . . .  
}
```

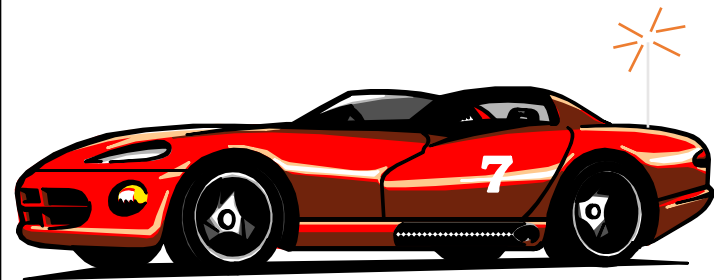
When we activate move, the computer signals the car to move forward one foot.



# The move Function

```
int main( )  
{  
    Car racer(7);  
  
    racer.turn_around( );  
    racer.move( );  
  
    . . .
```

When we activate move, the computer signals the car to move forward one foot.

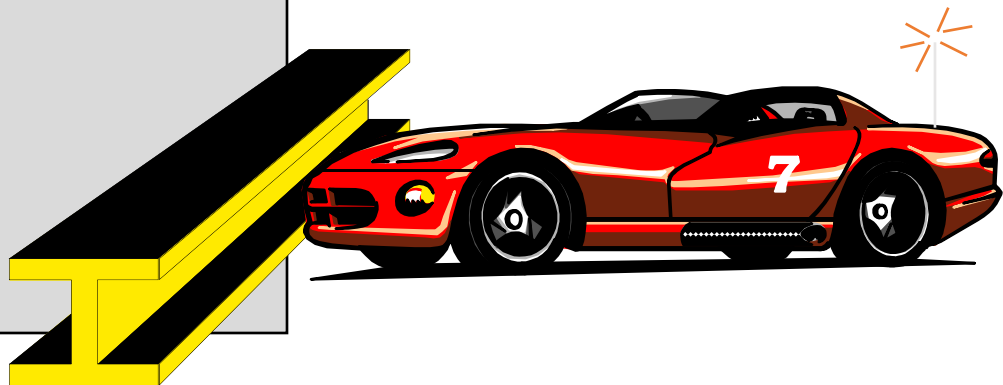




# The is\_blocked( ) Function

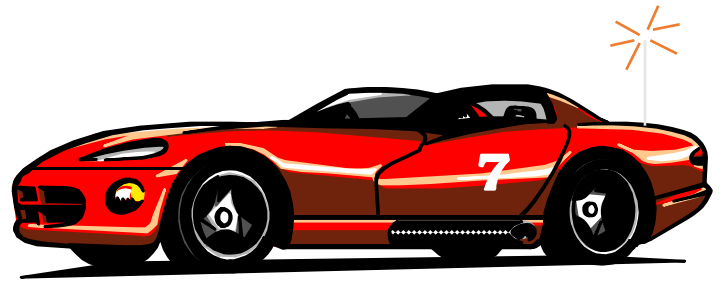
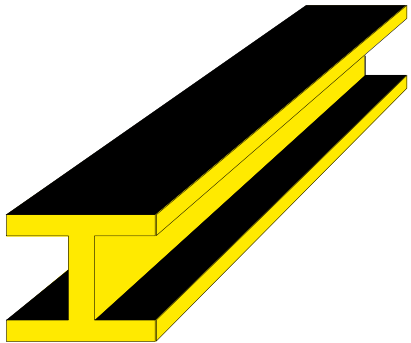
```
int main( )  
{  
    Car racer(7);  
  
    racer.turn_around( );  
    racer.move( );  
    if (racer.is_blocked( ))  
        cout << "Cannot move!";  
    . . .  
}
```

The is\_blocked member function detects barriers.



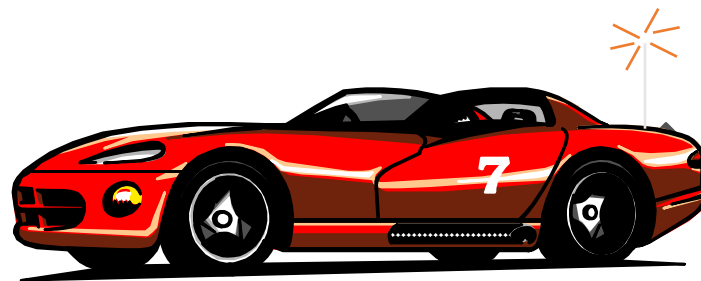
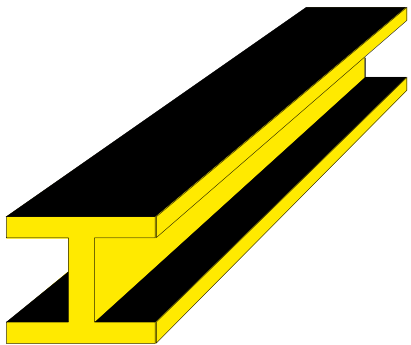
# Your Mission

- Write a function which will move a Car forward until it reaches a barrier...



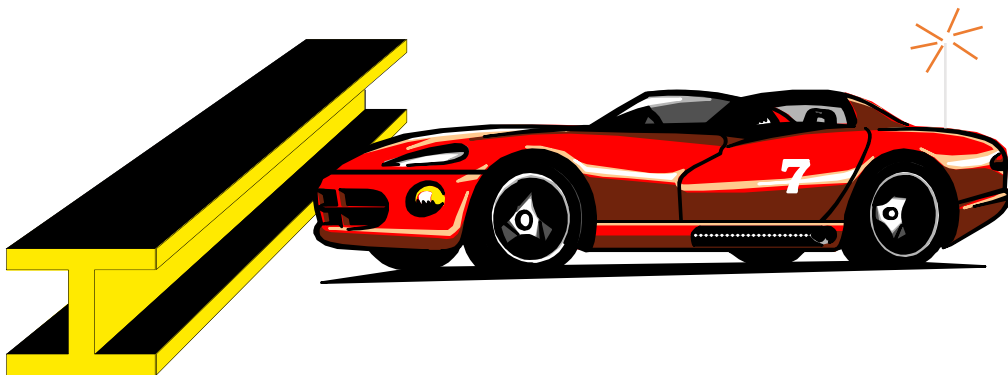
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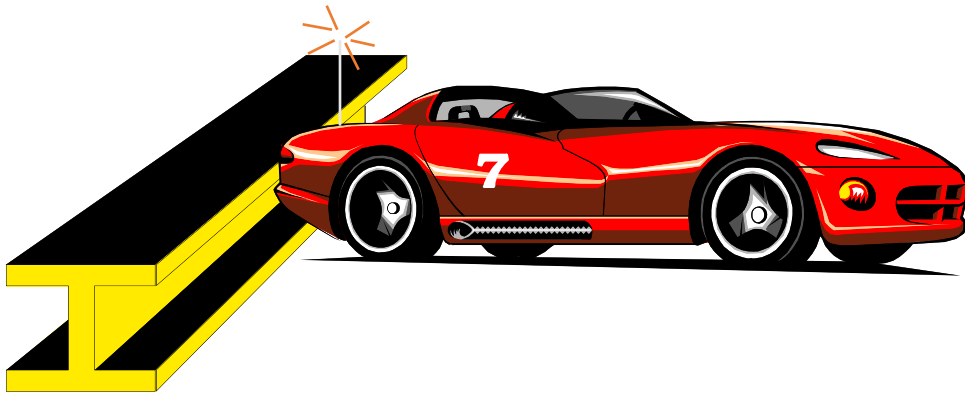
# Your Mission

- Write a function which will move a Car forward until it reaches a barrier...



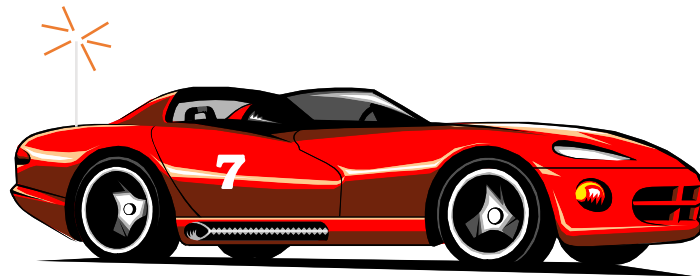
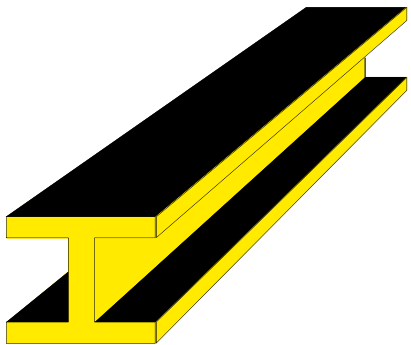
# Your Mission

- Write a function which will move a Car forward until it reaches a barrier...
- ...then the car is turned around...



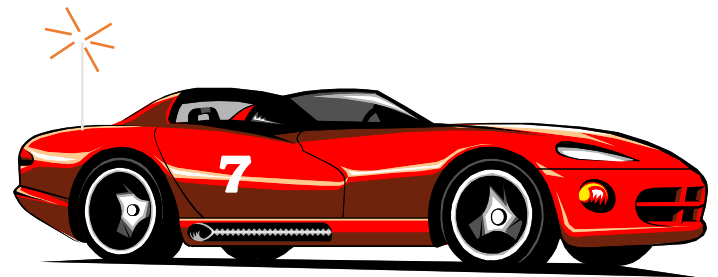
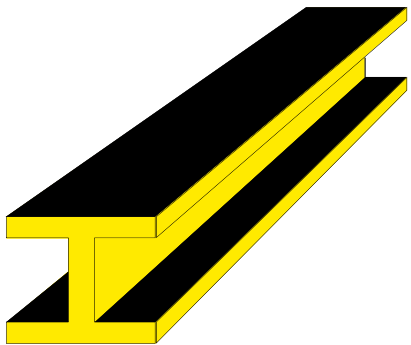
# Your Mission

- Write a function which will move a Car forward until it reaches a barrier...
- ...then the car is turned around...
- ...and returned to its original location, facing the opposite way.



# Your Mission

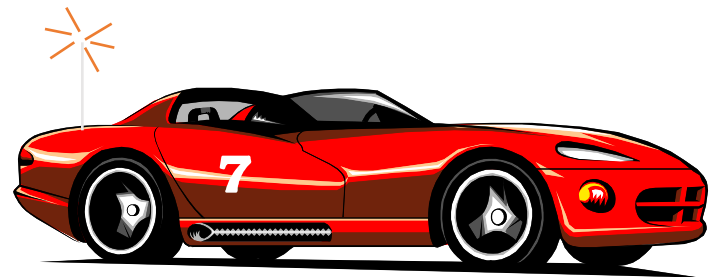
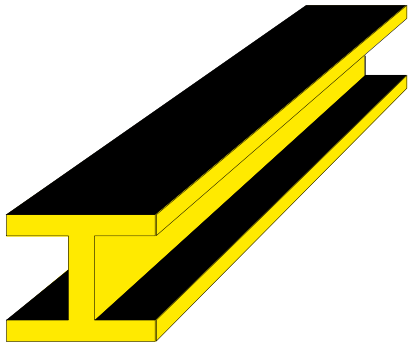
- Write a function which will move a Car forward until it reaches a barrier...
- ...then the car is turned around...
- ...and returned to its original location, facing the opposite way.



# Your Mission

```
void ricochet(Car& moving_car);
```

- Write a function which will move a Car forward until it reaches a barrier...
- ...then the car is turned around...
- ...and returned to its original location, facing the opposite way.





# Pseudocode for ricochet

```
void ricochet(Car& moving_car);
```

- ❶ if `moving_car.is_blocked( )`, then the car is already at the barrier. In this case, just turn the car around.

# Pseudocode for ricochet

```
void ricochet(Car& moving_car);
```

- ❶ if `moving_car.is_blocked( )`, then the car is already at the barrier. In this case, just turn the car around.
- ❷ Otherwise, the car has not yet reached the barrier, so start with:

```
moving_car.move( );
```

```
...
```

# Pseudocode for ricochet

```
void ricochet(Car& moving_car);
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```
moving_car.move( );
```

```
...
```

This makes the problem a bit **smaller**. For example, if the car started 100 feet from the barrier...

**100 ft.**



# Pseudocode for ricochet

```
void ricochet(Car& moving_car);
```

- ❶ if `moving_car.is_blocked( )`, then the car is already at the barrier. In this case, just turn the car around.
- ❷ Otherwise, the car has not yet reached the barrier, so start with:

```
moving_car.move( );
```

```
...
```

This makes the problem a bit **smaller**. For example, if the car started 100 feet from the barrier... then after activating `move` once, the distance is only 99 feet.



99



# Pseudocode for ricochet

```
void ricochet(Car& moving_car);
```

- ❶ if `moving_car.is_blocked( )`, then the car is already at the barrier. In this case, just turn the car around.
- ❷ Otherwise, the car has not yet

```
moving_car.move( );
```

```
...
```

We now have a smaller version of the same problem that we started with.

99 ft.



# Pseudocode for ricochet

```
void ricochet(Car& moving_car);
```

- ❶ if `moving_car.is_blocked( )`, then the car is already at the barrier. In this case, just turn the car around.
- ❷ Otherwise, the car has not yet reached the barrier, so start with:

```
moving_car.move( );  
ricochet(moving_car);  
...
```

Make a recursive call to solve the smaller problem.

99 ft.



# Pseudocode for ricochet

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- ❶ if `moving_car.is_blocked( )`, then the car is already at the barrier. In this case, just turn the car around.
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moving_car.move( );  
ricochet(moving_car);  
...
```

The recursive call  
will solve the  
smaller problem.

99 ft.



# Pseudocode for ricochet

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void ricochet(Car& moving_car);
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moving_car.move( );  
ricochet(moving_car);  
...
```



The recursive call  
will solve the  
smaller problem.





# Pseudocode for ricochet

```
void ricochet(Car& moving_car)
```

- ① if moving\_car.is\_blocked( ),  
this case, just turn the car a
- ② Otherwise, the car has not y

```
moving_car.move( );  
ricochet(moving_car);  
...
```

The recursive call  
will solve the  
smaller problem.



# Pseudocode for ricochet

```
void ricochet(Car& moving_car)
```

- ① if moving\_car.is\_blocked( ),  
this case, just turn the car a
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```
moving_car.move( );  
ricochet(moving_car);  
...
```

The recursive call  
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smaller problem.



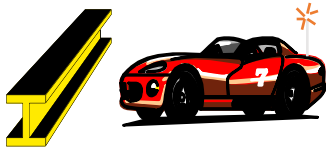
# Pseudocode for ricochet

```
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- ❷ Otherwise, the car has not yet

```
moving_car.move( );  
ricochet(moving_car);  
...
```

The recursive call will solve the smaller problem.



# Pseudocode for ricochet

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- ❷ Otherwise, the car has not yet

```
moving_car.move( );  
ricochet(moving_car);  
...
```

The recursive call will solve the smaller problem.



# Pseudocode for ricochet

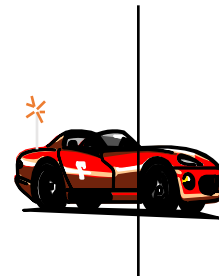
```
void ricochet(Car& moving_car)
```

- ① if moving\_car.is\_blocked( ),  
this case, just turn the car around
- ② Otherwise, the car has not yet

The recursive call will solve the smaller problem.

```
moving_car.move( );  
ricochet(moving_car);
```

...



# Pseudocode for ricochet

```
void ricochet(Car& moving_car)
```

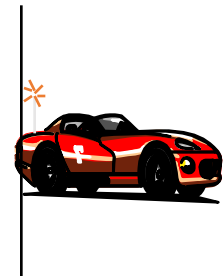
- ❶ if moving\_car.is\_blocked( ),  
this case, just turn the car around
- ❷ Otherwise, the car has not yet

The recursive call will solve the smaller problem.

```
moving_car.move( );  
ricochet(moving_car);  
...
```



99 ft.





# Pseudocode for ricochet

```
void ricochet(Car& moving_car)
```

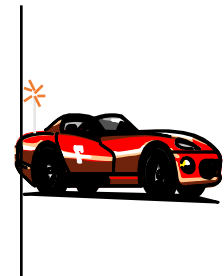
- ❶ if moving\_car.is\_blocked( ),  
this case, just turn the car around
- ❷ Otherwise, the car has not yet

*What is the last step  
that's needed to return  
to our original  
location?*

```
moving_car.move( );  
ricochet(moving_car);  
...
```



99 ft.



# Pseudocode for ricochet

```
void ricochet(Car& moving_car)
```

- 1 if moving\_car.is\_blocked( ),  
this case, just turn the car around
- 2 Otherwise, the car has not yet

*What is the last step  
that's needed to return  
to our original  
location?*

```
moving_car.move( );  
ricochet(moving_car);  
moving_car.move( );
```

100 ft.



# Pseudocode for ricochet

```
void ricochet(Car& moving_car);
```

- ❶ if `moving_car.is_blocked( )`, then the car is already at the barrier. In this case, just turn the car around.
- ❷ Otherwise, the car has not yet reached the barrier, so start with:

```
moving_car.move( );  
ricochet(moving_car);  
moving_car.move( );
```

This recursive function follows a common pattern that you should recognize.

# Pseudocode for ricochet

```
void ricochet(Car& moving_car);
```

- ① if `moving_car.is_blocked( )`, then the car is already at the barrier. In this case, just turn the car around.
- ② Otherwise, the car has not yet reached the barrier, so start with:

```
moving_car.move( );  
ricochet(moving_car);  
moving_car.move( );
```

When the problem is simple, solve it with no recursive call.

This is the **base case** or the **stopping case**.

# Pseudocode for ricochet

```
void ricochet(Car& moving_car);
```

- ❶ if `moving_car.is_blocked( )`, then the car is already at the barrier. In this case, just turn the car around.
- ❷ Otherwise, the car has not yet reached the barrier, so start with:

```
moving_car.move( );  
ricochet(moving_car);  
moving_car.move( );
```

When the problem is more complex, start by doing work to create a **smaller** version of the **same problem...**

# Pseudocode for ricochet

```
void ricochet(Car& moving_car);
```

- ❶ if `moving_car.is_blocked( )`, then the car is already at the barrier. In this case, just turn the car around.
- ❷ Otherwise, the car has not yet reached the barrier, so start with:

```
moving_car.move( );  
ricochet(moving_car);  
moving_car.move( );
```

...use a **recursive call** to completely solve the smaller problem...

# Pseudocode for ricochet

```
void ricochet(Car& moving_car);
```

- ❶ if `moving_car.is_blocked( )`, then the car is already at the barrier. In this case, just turn the car around.
- ❷ Otherwise, the car has not yet reached the barrier, so start with:

```
moving_car.move( );  
ricochet(moving_car);  
moving_car.move( );
```

...and finally do any work that's needed to complete the solution of the original problem..

# Implementation of ricochet

```
void ricochet(Car& moving_car)
{
    if (moving_car.is_blocked( ))
        moving_car.turn_around( ); // Base case
    else
    {
        // Recursive pattern
        moving_car.move( );
        ricochet(moving_car);
        moving_car.move( );
    }
}
```

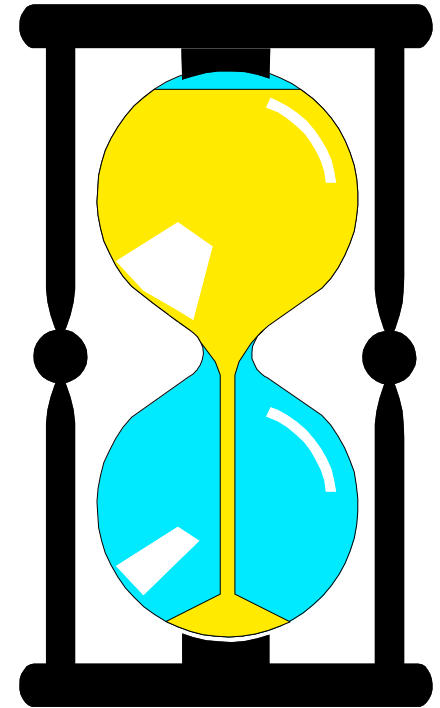
Look for this pattern in the other examples of Chapter 9.



# An Exercise

*Can you write `ricochet` as a new member function of the `Car` class, instead of a separate function?*

```
void Car::ricochet( )  
{  
    . . .
```



*You have 2 minutes to write the implementation.*

# An Exercise

One solution:

```
void Car::ricochet( )
{
    if (is_blocked( ))
        turn_around( ); // Base case
    else
    { // Recursive pattern
        move( );
        ricochet( );
        move( );
    }
}
```

# Recursive Thinking: General Form

- Recursive Calls
  - Suppose a problem has one or more cases in which some of the subtasks are simpler versions of the original problem. These subtasks can be solved by recursive calls
- Stopping Cases /Base Cases
  - A function that makes recursive calls must have one or more cases in which the entire computation is fulfilled without recursion. These cases are called stopping cases or base cases

# Tracing Recursive Calls: Ricochet

Do it by hand if car is 4 feet away from the barrier

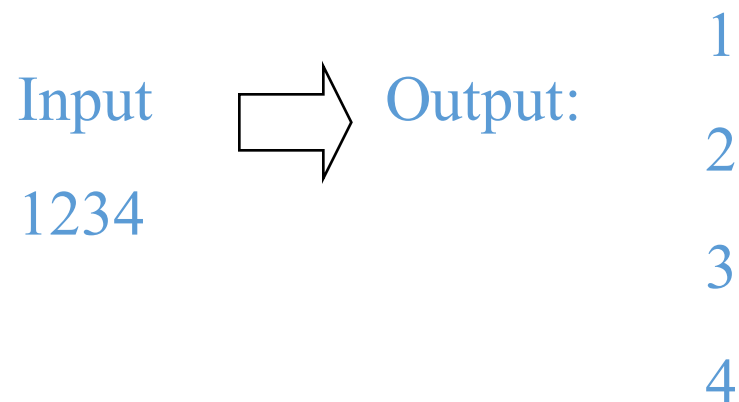
```
void Car::ricochet( )
{
    if (is_blocked( ))
    A.    turn_around( ); // Base case
    else
    {    // Recursive pattern
    B.    move( );
    C.    ricochet( );
    D.    move( );
    E }
}
```

# A Close Look at Ricochet Recursion

- The recursive case and the stopping case
- Activation record
  - The return location only in this example – other information is kept in the object racer
- The running stack
  - The collection of the activation records is stored in a stack data structure

# Example 2: Write Number Vertically

- Task
  - Write a non-negative integer to the screen with its decimal digits stacked vertically
  - for example:



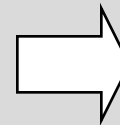
# A possible function

## Write an integer number vertically

```
void write_vertical (unsigned int number)
// precondition: number >=0
// Postcondition: The digits of number have been written, stacked vertically.
{ assert(number>=0);
  do
  {
    cout << number % 10 << endl;    // Write a digit
    number = number / 10;
  } while (number !=0);
}
```

Input

1234



Output:

4

3

2

1

# Approach 1: using a stack

## Write an integer number vertically

```
void stack_write_vertical (unsigned int number)
// Postcondition: The digits of number have been written, stacked vertically.
{
    stack<int> s;
    do
    {
        s.push(number % 10);    // push a digit in the stack
        number = number / 10;
    } while (number !=0);
    while (!(s.empty()))
    {
        cout << s.top()<< endl;    //print a digit from the stack
        s.pop();
    }
}
```



## Approach 2: Using Recursion

### Write an integer number vertically

```
void recursive_write_vertical(unsigned int number)
// Postcondition: The digits of number have been written, stacked vertically.
{
    if (number < 10) // stopping case
        cout << number << endl; // Write the one digit
    else // including recursive calls
    {
        recursive_write_vertical(number/10); // Write all but the last digit
        cout << number % 10 << endl; // Write the last digit
    }
}
```

# Tracing Recursive Calls

## Write an integer number vertically

```
void recursive_write_vertical_2(unsigned int number)
// Postcondition: The digits of number have been written, stacked vertically.
{
    if (number < 10) // stopping case
A cout << number << endl; // Write the one digit
    else // including recursive calls
    {
B recursive_write_vertical(number/10); // Write all but the last digit
C cout << number % 10 << endl; // Write the last digit
D }
}
```

# A Closer Look at the Recursion

- **Recursive Function**
  - Recursive calls
  - Stopping (Base) cases
- **Run-time Stack**
  - the collection of activation records is stored in the stack
- **Activation Record** - a special memory block including
  - return location of a function call
  - values of the formal parameters and local variables

# Recursive Thinking: General Form

- Recursive Calls
  - Suppose a problem has **one or more** cases in which some of the subtasks are simpler versions of the original problem. These subtasks can be solved by recursive calls
- Stopping Cases /Base Cases
  - A function that makes recursive calls must have **one or more** cases in which the entire computation is fulfilled without recursion. These cases are called stopping cases or base cases

# Self-Tests and More Complicated Examples

- An Extension of `write_vertical` (page 436)
  - handles all integers including negative ones
  - Hints: you can have more than one recursive calls or stopping cases in your recursive function
- Homework
  - Reading: Section 9.1
  - Self-Test: Exercises 1-8
  - Advanced Reading: Section 9.2
  - **Assignment 5 online, due Nov 9, 2016**

# super\_write\_vertical

## Write any integer number vertically

```
void super_write_vertical(int number)
// Postcondition: The digits of the number have been written, stacked vertically.
// If number is negative, then a negative sign appears on top.
// Library facilities used: iostream.h, math.h
{
    if (number < 0)
    {
        cout << '-' << endl;          // print a negative sign
        super_write_vertical(abs(number)); // abs computes absolute value
        // This is Spot #1 referred to in the text.
    }
    else if (number < 10)
        cout << number << endl;      // Write the one digit
    else
    {
        super_write_vertical(number/10); // Write all but the last digit
        // This is Spot #2 referred to in the text.
        cout << number % 10 << endl;   // Write the last digit
    }
}
```